Szent István University

ARCHING IN GRANULAR MATERIALS

Ph.D. theses

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Chapter 1

Scientific premises, objectives

Engineers working on the field of agriculture, pharmaceutical industries or architecture frequently meet with problems arising from the special properties of granular materials.

1.1 Importance of the subject

Definition 1. *Arching* means – according to the literature – the formation of a material layer in the granular assembly (caused by the load acting on it) which is capable to bear the load arising from the material above.

The appearance of arches on the one hand holds up the flow of granular material (e.g. the discharge of containers), and on the other hand results overload on the container walls, sometimes causing the collapse of the container.

1.2 Scientific premises

Definition 2. *Critical outlet width* means that smallest outlet size, which is wide enough to let the granular material out of it's container without obstruction.

Scientists trying to describe the arching phenomenon were trying to answer two important questions.

1. To evaluate the stresses causing the compressive stresses, because the compaction of granular media gives the capability for the material to bear it's own weight and the load arising from the material above it.

2. To evaluate the stresses in the neighborhood of the arches, because these stresses cause the collapse of the arch, and by this way determine the critical outlet width.

The values of critical outlet width determined using the methods found in the literature are in many cases more than two times larger than the measured ones.

The new arching model developed in my dissertation deals with the arching process using an entirely different approach to the phenomenon.

1.3 Objectives

I created a new model of the arching phenomena. With the use of this new model, my estimations were in better correlation with the measurements, than the results found in the literature. In my dissertation I:

1. presented the material and failure criteria used in the continuum model of granular assemblies,
2. summarized the results found in the literature dealing with the stresses arising in granular assemblies and the modelling of the arching phenomena,
3. presented the measurement methods and apparatuses used for the determination of the material- and failure properties,
4. presented my new arching examining apparatus and my modifications on the triaxial apparatus,
5. introduced a new model of the arch formation and collapse, using the results of my arching measurements,
6. introduced my results in the numerical simulation of the arching phenomena, using my new model.

Chapter 2

Experiments

The aim of my experimental studies were the determination of material- and failure properties of granular materials.

2.1 Measurement of material properties

I used a special triaxial apparatus developed at our faculty for the determination of material properties. Using this apparatus, I determined thew granular material's Young modulus and it's Poisson's ratio. For the finite element model of the granular media, I used the stress-strain characteristics of the material.

2.2 Measurement of the failure properties

Analyzing the arch formation and collapse process I found, that two failure properties have to be applied for the proper description of the arching process.

2.2.1 Shear failure

Definition 3. (Mohr–Coulomb criterion.) Within the granular material there shear failure occurs, if somewhere can be found a plane, where the shear stress overruns a specified ratio of the normal stress.

For the determination of the shear failure function, I used a shear box used in soil mechanics, modified by *Balássy*.

Analyzing the failure process of arches, I found, that the collapse of the arch always begins with ”crack” propagation. To describe this crack propagation process, I evaluated the deformation energy density arising in the shear zone of the shear box.

2.2.2 Failure in biaxial stress state

The failure properties of granular materials are not only depending from the compressing stresses, but also from the stress state. Granular materials in biaxial stress state are capable to bear only significantly smaller value of compressing stresses than in triaxial state.

Definition 4. *Failure in biaxial stress state* means that a material element loses it's strength because of the stresses acting on it.

In the neighborhood of the arches free surface, there the granular material is in biaxial stress state.

To measure the critical compressive stress in biaxial state, I applied two different kind of load on the granular material. In the first step a triaxial pre-compression on the specimen, then I removed one of the lateral springs, and increased the vertical load until the collapse of the specimen. With the removal of the lateral spring, the biaxial stress state has been realized.

From the results of this measurements, it was possible to construct the materials failure function in biaxial stress state.

2.2.3 Arching experiments

For the examination of the arching process I developed an experimental apparatus to analyze the arching action in granular assemblies.

In the arching examiner apparatus I loaded the granular sample with different horizontal and vertical pressures. After loading, the outlet size was changed until the formation of the first stable arch. The formation of arches was videotaped. Then the outlet size was enlarged, until the collapse of the arch. The whole arch formation and collapse process was videotaped, then the process was fully analyzed.

Statement 1. It became clear from the experiments, that materials without cohesion are unable to form arches. Because of this, the cohesion is an important material property from the arching point of view

Statement 2. Using the results of measurements, I determined the parameters useful to characterize the arches: the arch height-width ratio and the maximal arch width.

Statement 3. Using measurements I proved, that the shape of arches can be approximated with parabola.

According to the videotaped arching experiments, the arches were taking form always because of the falling out of material from the middle of the material above the outlet. The arches were quite stable. The ”cracks” causing the collapse were always arising in the abutment of the arch.

After the first stable arch has been taking form, I started to enlarge the outlet width. Because of this, the arches became bigger and bigger until the last stable arch has been taking form. The outlet size belonging to the last stable arch I called critical outlet width.

Chapter 3

Model of the arching phenomenon

Statement 4. Only the critical outlet size can be used to determine the usefulness of any arching theory.

My studies showed, that it is impossible to take out from the granular material only the neighborhood of the arch. I had to change the definition of arching.

Definition 5. *Arching* means an equilibrium state of the whole assembly, when the granular material does not flow out from the container through the open outlet. The formation of an arch is an equilibrium state evolving as a result of the interaction between the container wall and the continuum elements of the granular assembly.

3.1 Arching in flat bottomed bins

To describe the arching action as an equilibrium state for the whole assembly, the simplest model was a flat bottomed bin, with a changeable outlet in it's bottom. The method developed for this type of a bin can be easily changed later to the case of hoppers.

3.1.1 Arch formation

To evaluate the stresses arising in the granular media, I created a simple mechanical model of a bin. From the mathematical point of view, the determination of stresses is the solution of the equation system of elasticity with proper boundary conditions:

$$\boldsymbol{F} \cdot \nabla + \mathbf{f} = \mathbf{0}, \tag{3.1}$$
$$\frac{1}{2}\left(\mathbf{u} \circ \nabla + \nabla \circ \mathbf{u}\right) = \boldsymbol{A}, \tag{3.2}$$
$$\boldsymbol{C} \cdot \boldsymbol{A} = \boldsymbol{F}, \tag{3.3}$$
$$\mathbf{u}|_{A_u} = \mathbf{u}_0, \tag{3.4}$$
$$\boldsymbol{F} \cdot \mathbf{n}|_{A_p} = \mathbf{p}_0. \tag{3.5}$$

The boundary conditions:

1. $(u_x = 0)$ at the vertical walls .
2. $(u_y = 0)$ at the bottom, in case of closed outlet.
3. The load at the top equals zero $(p_y = 0)$.

The base assumption for the arch formation is that the granular assemblies are unable to resist tension. This can be specified in the following mathematical form. First I had to determine the stress state in all points, with finite element method. Then I evaluated the eigenvalues of the stress tensor, the so called principal stresses in all points, using the

$$\left(\boldsymbol{F} - \sigma_n \boldsymbol{E}\right)\mathbf{n} = \mathbf{0} \tag{3.6}$$

equations. In those points of the granular mass, where the first (the biggest) principal stress is positive, there tension occurs.

All the points, where the first principal stress was positive (where tension occurred) were deleted from the finite element model. After the removal of these elements, I evaluated the stresses again. With the newly evaluated stresses field the principal stresses were evaluated again, then the failure criterion for biaxial stress state was checked in all boundary points.

Definition 6. *Specific stress* means $\frac{\sigma}{\rho g H}$ where H is the load height.

Examining the distribution of the first principal stresses over the open outlet I find, that at the middle of the model in a certain hight over the open outlet this principal stress changes from tension to compression.

All the points, where the first principal stress was positive (where tension occurred) were deleted from the finite element model. After the removal of these elements, I evaluated the stresses again. My finite element simulations showed, that the recurrence of these steps stops, and an arch takes form. This does not mean, that this arch is stable.

3.1.2 Arch collapse

Statement 5. In biaxial stress state, granular materials are unable to bear compressing stresses higher than a critical value. This critical value depends on the pre-compressing stresses. As long as the smallest principal stress is higher than the critical value, the arch does not collapse.

The outlet size can be extended until at the boundary of the stable arches the compressing stresses absolute value is smaller than the critical value.

Statement 6. According to my finite element simulations, the maximal compressing stress arises at the neighborhood of the abutment of the arch.

The analysis of the videotaped arch collapses showed that the cause of the arch collapse is not only the implosion of the material element which is in critical biaxial stress state. Such an implosion causes only the formation of a new arch. For the collapse of the *entire* material assembly the starting of crack propagation is also necessary.

Theorem 1. *Necessary and sufficient condition for arch collapse.*

1. *For the collapse of an arch in one of the material elements at the neighborhood of the arch – where the material elements are in biaxial stress state – the compressing stress must be higher than the critical value.*

2. *In the same element, the specific deformation energy must be higher than a critical value.*

In that case, when all the two conditions are met, than the arches are collapsing, and the material flows out from its container.

3.1.3 Arching algorithm

Definition 7. The following procedure is called *arching algorithm*:

1. Designate the T range of our interest, define the material- and failure properties:
   - the ρ density,
   - the E Young modulus,
   - the ν Poisson coefficient and the
   - yield function $\sigma_K(\sigma_t)$ belonging to biaxial stress state.

2. Define the boundary conditions. The opening of the outlet can be modeled here.

3. Solve the equations of elasticity with the previously defined boundary conditions.

$$\boldsymbol{F} \cdot \nabla + \mathbf{f} = \mathbf{0}, \tag{3.7}$$

$$\frac{1}{2} (\mathbf{u} \circ \nabla + \nabla \circ \mathbf{u}) = \boldsymbol{A}, \tag{3.8}$$

$$\boldsymbol{C} \cdot \boldsymbol{A} = \boldsymbol{F}, \tag{3.9}$$

$$\mathbf{u}|_{A_u} = \mathbf{u}_0, \tag{3.10}$$

$$\boldsymbol{F} \cdot \mathbf{n}|_{A_p} = \mathbf{p}_0. \tag{3.11}$$

4. Evaluate the principal stresses using the

$$(\boldsymbol{F} - \sigma_n \boldsymbol{E})\, \mathbf{n} = \mathbf{0} \tag{3.12}$$

   equations.

5. Remove those material elements, where $\sigma_1 > 0$.

6. If $\sigma_3 > \sigma_K$ at the arches boundary then go to step 8. If $\sigma_3 \leq \sigma_K$ becomes true, than the continuum element collapses.

7. If the specific deformation values is higher than the critical u_K value, than instable crack propagation starts within the granular material, and the whole assembly collapses, and the material flows out of the container. If the specific deformation energy is smaller than the critical value, than we can go to the next step.

8. After the removal of the broken elements define the boundary conditions for the new T range, and go back to step 3.

Statement 7. According to my experiences, the algorithm can stop in three ways.

– The whole material flows out from the container because of tension.

– At some step the$\sigma_3 \leq \sigma_K$, condition becomes true. This also means the beginning of the outflow.

– In some cases, the tension ceases, and in the same time the $\sigma_3 > \sigma_K$ condition is true in the whole assembly. *This means the formation of stable arches within the granular assembly.*

3.1.4 Arching in the hopper

For the modelling of the arching process in the hopper, I created the finite element model of a silo.

Examining the third principal stress distribution at the silo wall in the neighborhood of the silo wall I found, that the third principal stress is significantly higher at the transition zone than anywhere else. And in the same time the this third principal stress always has a minimum value somewhere in the middle of the hopper.

Statement 8. The $\frac{\sigma_3}{\rho g H}$ specific third principal stress function has a global maximum value at the transition zone, and a local minimum in the neighborhood of the middle of the hopper.

Definition 8. The fraction of the minimal and maximal values of the specific third principal stress is called by me *stress ratio*.

Theorem 2. *Arches are taking form at the hopper, if the critical stress value σ_K (evaluated from the maximal principal stress σ_3) is higher, than the minimal third principal stress arising within the hopper.*

Statement 9. My finite element calculations showed, that the stress ratio becomes smaller, if the outlet width decreases.

Statement 10. The specific deformation energy density has a maximum at the neighborhood of the abutment of the arch.

Statement 11. The maximal deformation energy values increases, when the outlet width becomes larger.

Statement 12. There is a critical values of hopper half angle from the arching point of view. If the hopper half angle is higher than this critical values, than arches are always taking form.

The stability of these arches depends on the critical deformation energy value. If this values is higher than a critical one, than the arches always collapsing, and in this case thew outflow of the granular material is a cycle of arch formation and collapse.

Theorem 3. *Necessary and sufficient condition of arching in the hopper. In the hopper there arches are taking form, if*

1. *the stress ratio becomes higher than a critical value;*
2. *and the deformation energy density at the abutment of the arch is also higher than a critical value.*

I found that:

– the increase of the wall friction decreases-,

- the increase of density decreases-,

– the increase of the Young modulus increases- and

– the increase of the Poisson's ratio decreases

the granular material's ability for arching.

The values of critical outlet width evaluated using my arching model were always significantly closer to the measured values than the results evaluated using the methods found in the literature.

3.2 New scientific results

Using measurements and numerical simulations I analyzed the arching action in granular assemblies, gave the necessary and sufficient conditions for the formation of arches. With the use of measurements and numerical simulations I determined the mechanical condition of the arch collapse, using triaxial apparatus I defined a measurement method for the determination of the arching properties of granular materials. With the modification of the triaxial apparatus, I made it possible to take into account the mechanical impact of the neighboring material during the measurement of the mechanical and failure properties of the sample.

1. Using measurements and numerical simulations I proved that the homogenous, isotropic material model is capable to model the arching action in granular assemblies.

2. Solving the differential equations describing the mechanical phenomena, and carrying out measurements I proved, that the necessary condition for the arch formation in granular assemblies is to have positive eigenvalues over the open outlet in the stress tensor field arising in the assembly because of the load.

3. With the use of numerical simulations I determined the arching process in flat bottomed bins.

4. Using measurements and solving the differential equations describing the mechanical phenomenon I proved, that the necessary condition of the arch formation in hoppers is that the σ_K critical biaxial compressing stress evaluated at the

transition zone from σ_3 maximal pre-compressing stress must be higher, than the minimal third main stress evaluated at the hopper vall.

5. With the use of numerical simulations, I determined the arching process at the hopper.

6. Using measurements I proved, that the shape of arches can be approximated with parabola.

7. With the use of numerical simulations I proved, that the stability of arches can be characterized using the value of specific deformation energy accumulated at the arch basement.

8. Using measurements and numerical simulations I proved, that the necessary condition of arch collapse (in case of flat bottomed bins and also in case of hoppers) is that the smallest eigenvalue of the stress field at the arch basement have to overrun a critical value, which value can be evaluated with measurements.

9. I developed a measurement method for the evaluation of the critical compressing stress belonging to biaxial stress state with triaxial apparatus.

Chapter 4

Summary

Arching means the formation of a material layer in the granular assembly – caused by the load acting on it – which is capable to bear the load arising from the material above.

The appearance of arches on the one hand holds up the flow of granular material (e.g. the discharge of containers), and on the other hand results overload on the container walls, sometimes causing the collapse of the container.

4.1 Summary of the research activity

In my dissertation I pointed out, that it is more appropriate to treat the arching phenomenon as an equilibrium state of the whole assembly, and because of this I gave a new definition of arching.

In my definition *arching* means an equilibrium state of the whole assembly, when the granular material does not flow out from the container through the open outlet. The formation of an arch is an equilibrium state evolving as a result of the interaction between the container wall and the continuum elements of the granular assembly.

I created a new model of the arching phenomena. With the use of this new model, my estimations were in better correlation with the measurements, than the results found in the literature. In my dissertation I:

1. presented the material and failure criteria used in the continuum model of granular assemblies,

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5. introduced a new model of the arch formation and collapse, using the results of my arching measurements,

6. introduced my results in the numerical simulation of the arching phenomena, using my new model.

4.2 New scientific results

Using measurements and numerical simulations I analyzed the arching action in granular assemblies, gave the necessary and sufficient conditions for the formation of arches. With the use of measurements and numerical simulations I determined the mechanical condition of the arch collapse, using triaxial apparatus I defined a measurement method for the determination of the arching properties of granular materials. With the modification of the triaxial apparatus, I made it possible to take into account the mechanical impact of the neighboring material during the measurement of the mechanical and failure properties of the sample.

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5. With the use of numerical simulations, I determined the arching process at the hopper.
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Chapter 5

Publications

1. **Keppler István**: *Szemcsés anyagok természetes boltozódása.* GÉP, 2006. I. p. 29-33.

2. **Keppler István**: *Boltozódásvizsgálatok.* MTA-AMB XXII. Kutatási és Fejlesztési Tanácskozás, Gödöllő, 1998.

3. Csizmadia B. - Csorba L. - **Keppler I.** : *Természetes boltozódás kísérleti és numerikus modellezése.* VIII. Magyar Mechanikai Konferencia 1999. Miskolc. p. 19.

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5. Csizmadia B. - **Keppler I.**: Néhány gondolat a szemcsés anyagok természetes boltozódásának modellezési lehetőségeiről. Fiatal Műszakiak Tudományos ülésszaka, Kolozsvár, 2000. március 24-25. FMTÜ 2000 p. 41-44.

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7. Csizmadia B. - **Keppler István**: *Mechanics of granular materials. Arching theories and experiments.* International multidisciplinary conference. May 23 - 24 2003 Baia Mare, Romania North University of Baia Mare Scientific bulletin Serie C, Volume XVII, Part II. p. 103 - 108.

8. Csizmadia B. - Oldal I. - **Keppler I.**: *Quasi-Triaxial apparatus for the determination of mechanical properties of granular materials.* 20th. Danubia - Adria Symposium on Experimental Methods in Solid Mechanics, September 24 - 27, 2003 Győr, Hungary.

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10. **Keppler István**: *Természetes boltozatok kialakulásának és tönkremenetelének végeselem modellje.* Fiatal Műszakiak Tudományos ülésszaka, Kolozsvár, 2005. március 18-19. FMTÜ 2005.

11. **Keppler István**: *Finite element simulation of arching in granular assemblies.* 4th YSESM, Castrocaro Terme, Italy, 2005. 05. 07.

12. **Keppler István**: *Mathematical modelling of arch formation in granular materials.* International multidisciplinary conference. 2005 Baia Mare, Romania.

13. **Keppler István**: *Szemcsés halmazok Mohr-Coulomb féle nyírási tönkremenetelének elemzése.* Fiatal Műszakiak Tudományos ülésszaka, Kolozsvár, 2006. március 24-25.

14. **Keppler István**: *Finite element model of arching in silos.* 5th YSESM, Púchov, Slovakia, 2006. 05. 10.