Szent István University

FLOWING AND ARCHING PROPERTIES OF GRANULAR MATERIALS

Theses of PhD work

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Content

1. INTRODUCTION, AIM OF WORK

1.1. Introduction

In agricultural and food industry granular mateials (wheet, corn, barley, etc.) have to be stored and moved. Storing in silos gives many advantages but there are a few special problems. Discharge is an important question when granular materials are stored in a silo. In my work I examined discharge of granular materials from silos. There are two method of determining rate of discharge granular materials from silo. Johanson's method is theoretical Beverloo's one is empirical. The aim of our researching is creating a theoretical model that gives better results for discharge rate than the previouses. During my researcing a theoretical model was made which based on phenomena of arching during discharging. This assumption was confirmed by experiments. Next step was determined shape of arch by theoretical and experimental way. Results of them I used to determine velocity of flow and discharge rate. Then experiment was done in silo models. Evaluation of experiments my model was verified and borders of validity was determined.

1.2. Aim of work

1.2.1. Arching

Knowing shape of arch is neccecary for my discharge model. Since now researchers have used a few function for describing arches. These functions were good for solving problems with little deviation. There are two common used shape circular and parabolic. In this work these and a trigonometric shape are examined and compared.

1.2.2. Discharging

There are two method of determining rate of discharge granular materials from silo. Johanson's method is theoretical Beverloo's one is empirical. Limits of validity and accuracy of these models are examined. Aim of my work is creating a theoretical model that better of these.

3. EXPERIMENTAL METHODS

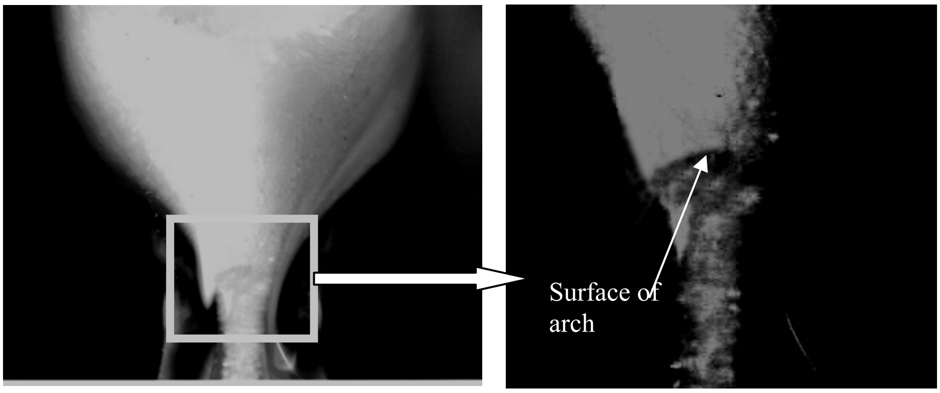
3.1. Examining of instable arches

3.1.1. Method

Arching of granular materials used to be examined when it stops discharging. But arching effect works in case of flowing out. It is called instable arch. During discharging instable arches are formatted and broken continually.
Phenomena can be confirmed by experiments. It can be seen if wall of container is transparent and outlet diameter is small enough. It can be achieved in case of sand-glass. Flowing in sand-glass is photographed with high speed.

3.1.2. Results

In Fig. 1.a) and b) an instable arch can be seen during flowing.



a) b)

Fig. 1. Instable arc during flowing out

I suppose that instable arches continually breaking and falling out gives discharge rate. This model is confirmed by this experiment. And results of this model to predict discharge rate confirm it, too

In case of fluids flowing velocity and discharge rate change with fluid level. In case of granular materials velocity is constant in function of filling level of silo. I suppose instable arches cause this phenomena. Predicting value of velocity knowing shape of arch is needed.

3.2. Measuring of shape of arch

3.3.1. Measuring equipment

In this equipment (Fig. 2.) a plane model of arch can be generated. Granular material is between two transparent plane walls with opened top. After filling and loading a narrow square outlet is opened at bottom. Below this slot an arch is formatted in plane strain conditions. Shape of arch can be seen and recorded at wall.

Fig. 2. Arching equipment

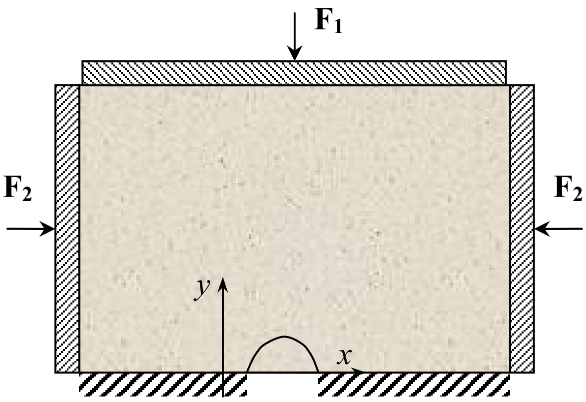
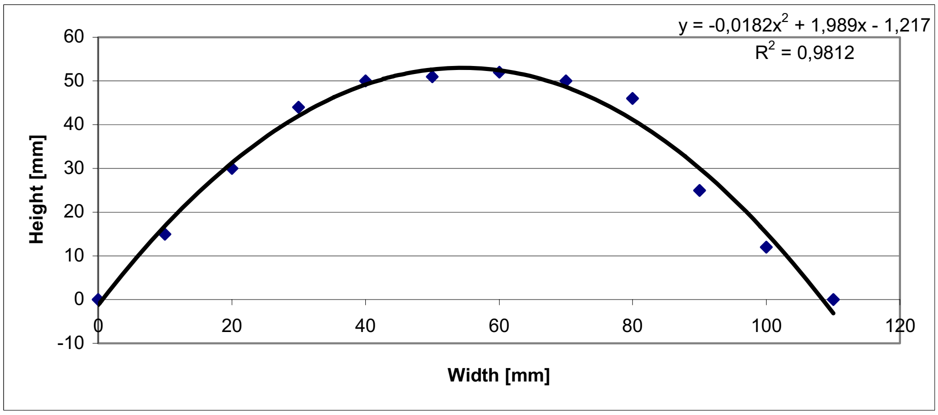


Fig. 3. Drawing of arching equipment

3.3.2. Results

Measured shapes of arches can be seen at Fig. 4. Quadratic function gives good approximation of shape.



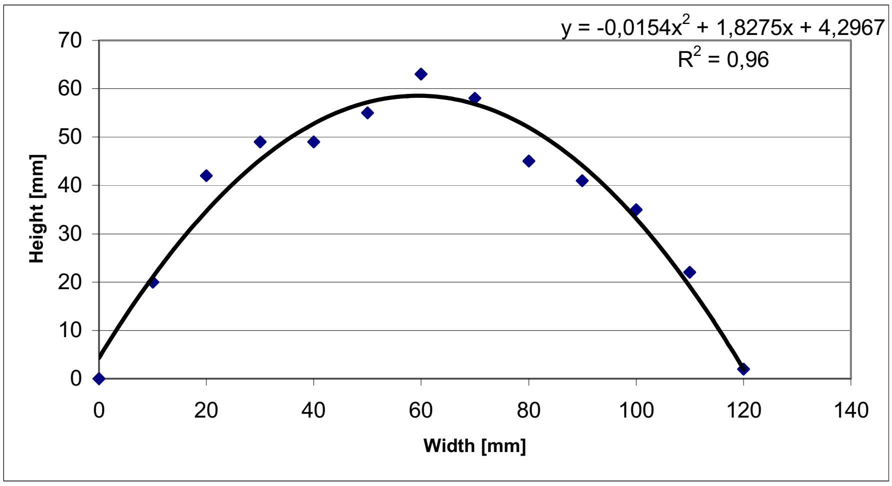


Fig. 4. Measured shapes of arches

3.4. Big silo model

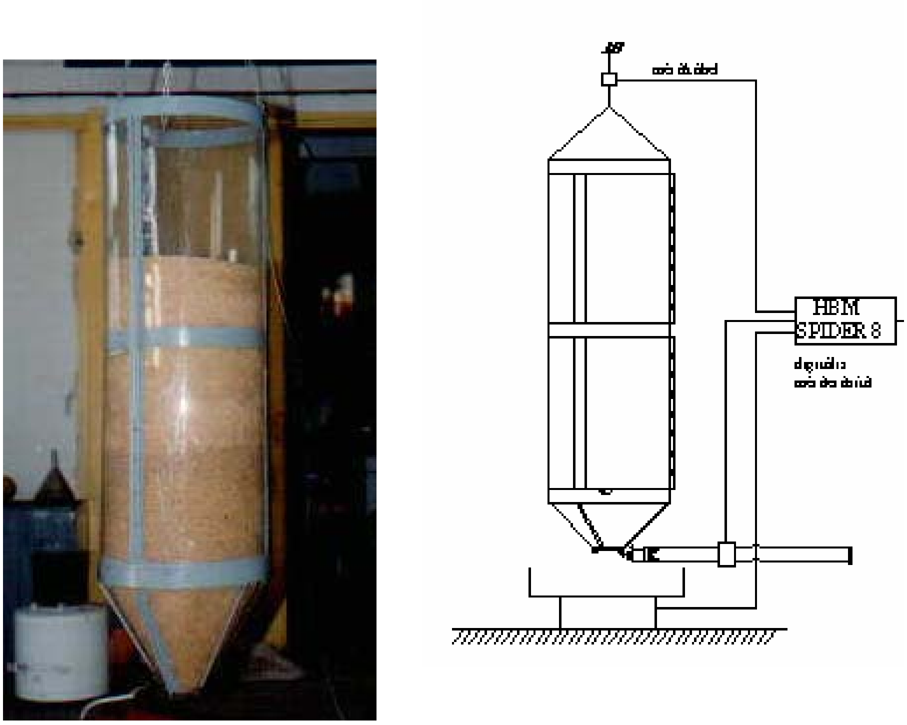


Fig. 5. Silo model

In Fig. 5. can be seen a big silo model. Height of model 1250 mm diameter 440 mm. Volume is 0.18 m^3 hopper is changeable. Outlet diameter is changeable diameter 100 mm and 50 mm. Half cone angles are 30°, 45°, 60°. Flowed material can be measured in function of time with computer.

3.5. Little silo model

Little silo model was made of a plastic cylinder of 100 mm in diameter. The hopper was conical and its half cone-angle was 45°. The outlet diameter was variable between 25 and 100 mm. The loads were measured by two load cells wall friction, vertical force on hopper sum of them was mass of material in silo. Experiments were done using grain wheat.
Aim of experiments was examining effect of changing geometry. First outlet size was changed next cone angle was changed.

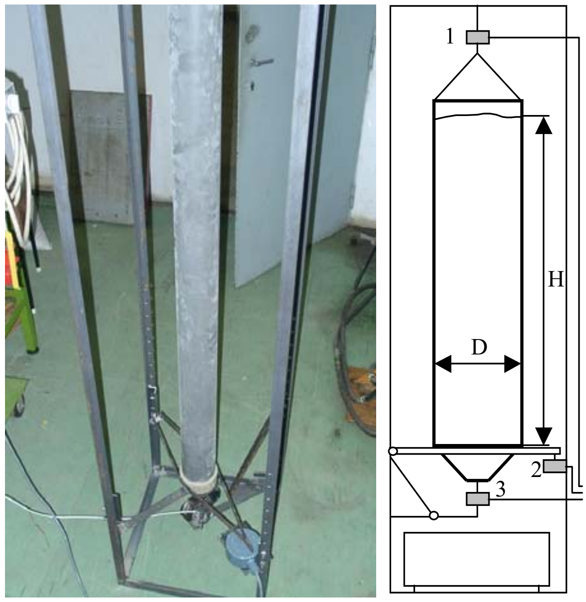


Fig. 6. Silo model

3.5.1. Measuring of velocity distribution

Method of measuring distribution of velocity is dividing the flow of granular material into two parts. Baffle is used with predetermined *b* size of slot (Fig. 7.). And two discharge rates are measured. Average velocity flow out certain areas is computed fom these datas. Diameter of outlet is 31 mm, size of slot is increased from 2 mm to half of diameter with 1 mm steps. Examined material is poppy seeds. Experiments are repeated three times with every settings.

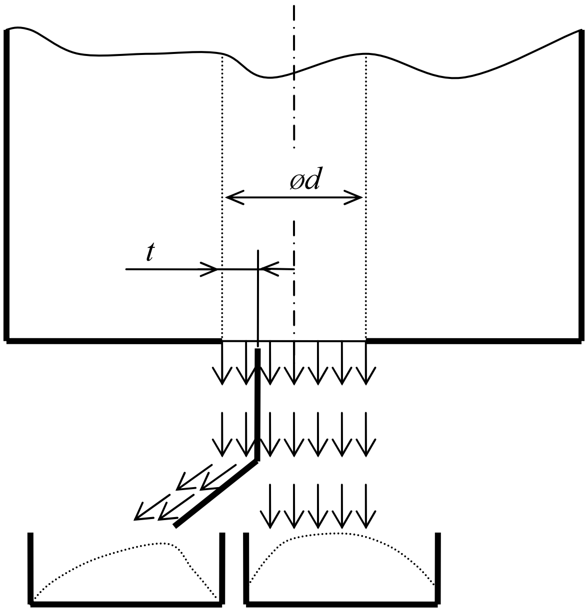


Fig. 7. Measuring of velocity distribution

Results of measuing are mass of material and flowing time in function of *b* szie of slot.

4. THEORETICAL RESEARCHING

4.1. Variation calculus for finding shape of arch

4.1.1. Variation problem

Function that describes shape of arch is found using similar method to Lagrange variation method.

4.1.2. Functional

This case density of total potential energy is the functional.

$$I = u(f(x)) = \frac{E_p(f(x))}{V(f(x))}$$

where:

- *f(x)* function of shape of arch,
- *u* density of energy,
- E_p potential energy,
- *V* volume of part,
- $n \in N$, element number of FEM.

4.1.3. A solution

We finded a numerical solution for this problem. Three type of functions were used for variation calculus. Solution was that given less functional.

Functional of functions:

Circle: I_{cir} = 1,4425.

Cosinus: I_{cos} = 1,4213.

Quadratic: I_{par} = 1,4216.

Functionals to functions were compared and it was minimalized by quadratic function. So parabolic shape is solution that gives good agreement with measuring.

4.2. Predicting of discharge rate

4.3.1. Theoretical model for funnel flow

We model the granular material flow out like that is forming and breaking of instable arches. That is continous if the conditions of stable arching are ungratified. Granular material is in conditions of free fall below arch. Flow velocity at outlet depends on height of falling only. So the first requirement is satisfied by the model namely discharge velocity is constant over height of bulk.

In order to determining value of mass flow we have to calculate velocity. It depends on height of arch

4.3.2. Velocity of granular material at outlet

Velocity is determined by height and shape of arch and starting velocity. In case of funnel flow starting velocity is zero so free falling computation method can be used.
Falling started from surface of arch that is described by $f(x,\varphi)$. It is a function of paraboloid (Fig. 8.) in polar coordinate system.

$$f(x,\varphi) = h\left(1-\left(\frac{2x}{d}\right)^2\right)$$

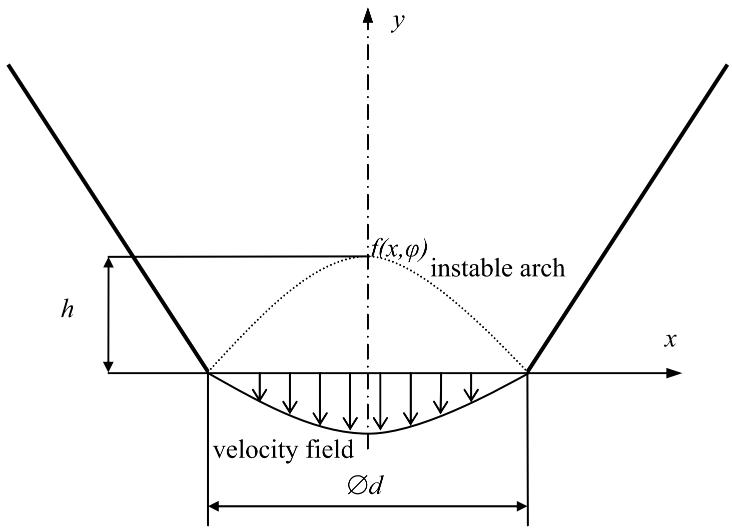


Fig. 8. Outlet of silo

Velocity in case of free falling:

$$v = \sqrt{2gh}\ .$$

Using function of surface:

$$v(x,\varphi) = \sqrt{2g \cdot h\left(1 - \left(\frac{2x}{d}\right)^2\right)}$$

Height of instable arch can not be measured. δ shape coefficient of arch was introduced: δ = *h/d*, rate of height and width of arch. Substituting and ordering:

$$v(x,\varphi) = \sqrt{2g\delta d}\sqrt{\left(1 - \left(\frac{2x}{d}\right)^2\right)}$$

Average velocity:

$$v = \frac{\int_0^{\frac{d}{2}} \int_0^{2\pi} \sqrt{2g\delta d} \sqrt{\left(1 - \left(\frac{2x}{d}\right)^2\right)} x d\varphi dx}{\frac{d^2\pi}{4}} = \frac{2}{3}\sqrt{2g\delta d}$$

4.3.3. Discharge rate

Using average velocity and bulk density discharge rate can be determined. Particles cause declining of outlet size so diameter is reduced with particle size.

$$W = \frac{\pi\sqrt{2g}}{6}\sqrt{\delta} \cdot \rho \cdot (d - d_p)^{\frac{5}{2}}$$

It can be used in case of funnel flow. It can be used for mass flow if δ is calibrated for cone angle.

5. RESULTS

5.1. Measuring of δ

Computational model contains a δ coefficient that can be measured as material property. In 1. table these coefficients are shown. These are calculated from measured discharge rates at funnel flow (60°) and mass flow (30°).

1. table. δ coeficient to different materials

Material	Wheat		Maise		Oat		PE-LD	
Half cone angle	60°	30°	60°	30°	60°	30°	60°	30°
δ	**0,4**	0,5	**0,3**	0,55	**0,3**	0,55	**0,3**	0,35

5.2. Velocity distribution

Measured and calculated discharge rates at slot sizes are shown in Fig. 9. Using constant velocity as Johanson and Beverloo discharge rates can be calculated from area of slot. In order to comparison discharge rate is calculated from our model:

$$W = \rho \int_{-\varphi}^{\varphi} \int_{\frac{R\cos(\varphi)}{\cos(\alpha)}}^{R} \sqrt{2g\delta d} \sqrt{\left(1 - \left(\frac{2x}{d}\right)^2\right)} x dx d\alpha$$

Where φ is angle of half circular arc with t slot size:

$$\varphi = \arccos\left(1 - \frac{t}{R}\right)$$

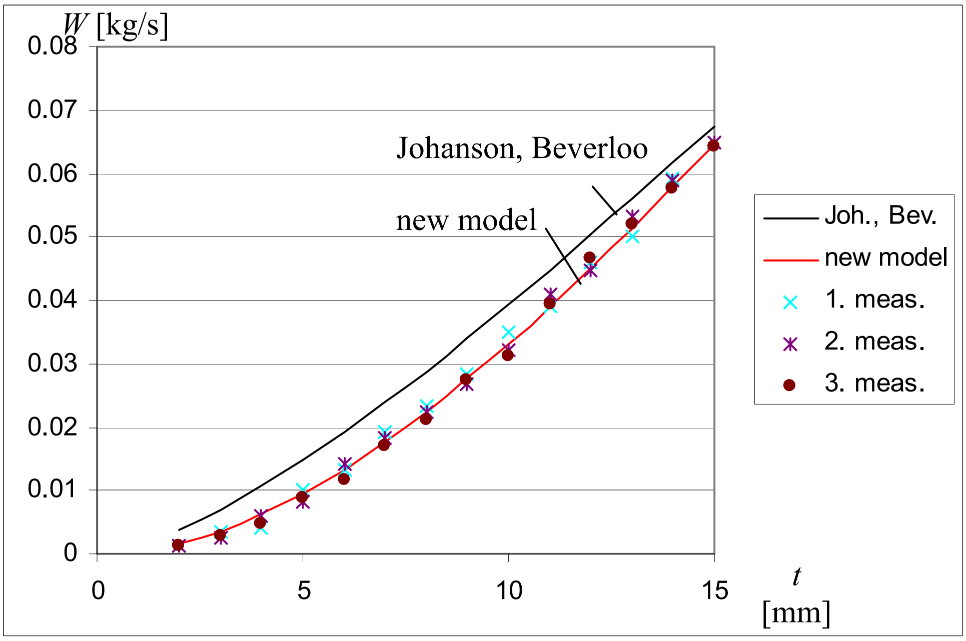


Fig. 9. Discharge rate in function of slot size

In Fig. 9. can be seen that measured discharge rates show good agreement with results of new model. We can make conclusion that new model gives better prediction of velocity distribution than the other models. Average velocity at 1 mm slots was calculating from discharge rates in order to examine velocity distribution. Comparison can be seen in Fig. 10.
Function of measured points is not known, so a cubic Taylor polynom was fit to them. But fitted curve is quadratic because of symmetry. Deviation of measured velocity is derived from deviation of slot adjustment.
After examination of results we find out new model shows very good agreement with results of measuring. So experiments verify that in contrast to assumption of other models outflow velocity is not constant along diameter of outlet.

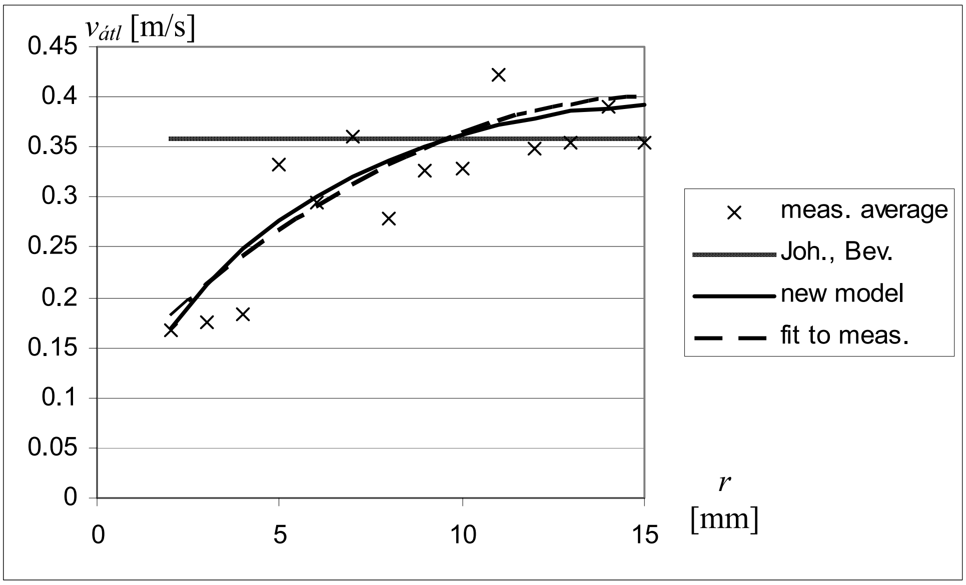


Fig. 10. Velocity in function of outlet radius

5.3. *Examining of outlet diameter*

Experiments were done with 25, 30, 35, 40, 45, 60, 70, 80, 90, 100 mm outlet diameters. Half cone angle was 45˚. In Fig. 11. can be seen results of measuring with big and small silos and computed discharge rates.

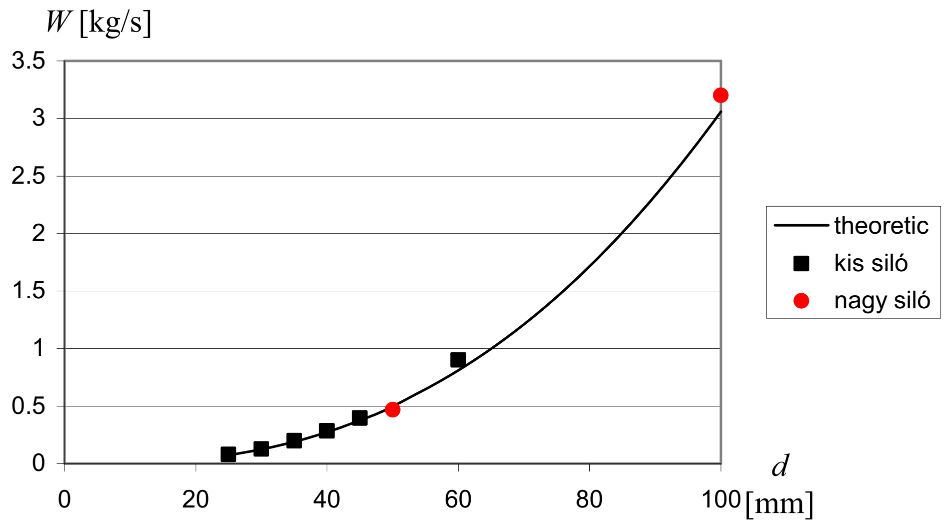


Fig. 11. Measured and predicted discharge rates

Our model gives good prediction for discharge rates. But when outlet size more than 60% of silo diameter makes more error in prediction (Fig. 12.). **So our model can be used below 60% of silo diameter.**

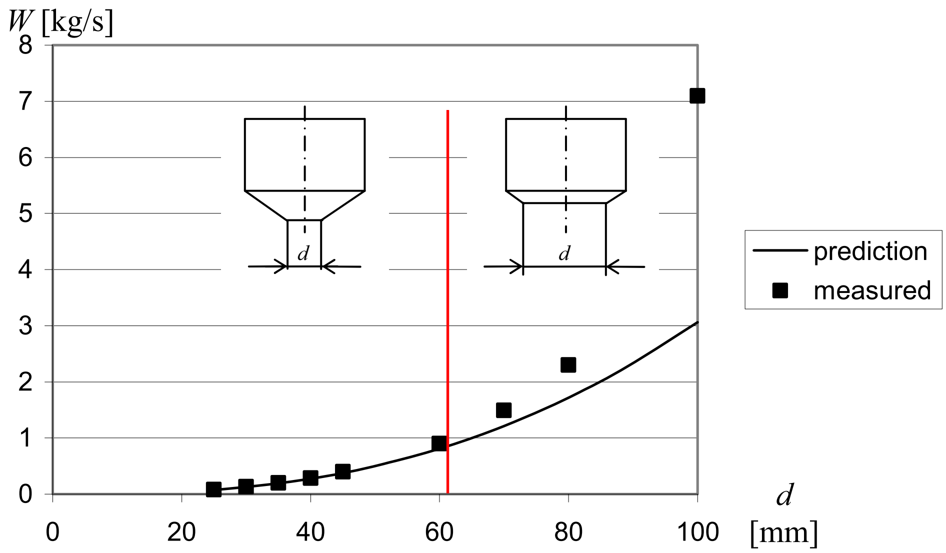


Fig. 12. Measured and predicted discharge rates

5.4. Examining of hopper angle

These expariments was done hoppers with 10, 20, 30, 37,5, 45, 90˚ half cone angle. Outlet diameter was 20 mm. Results are in Table 2. and Fig. 13.

Table 2.

Half cone angle [^{o}]	W [kg/s]	δ
10	0.077	1,96
20	0.067	1,2
30	0.038	0,62
37.5	0.031	0,4
45	0.031	0,4
90	0.03	0,4

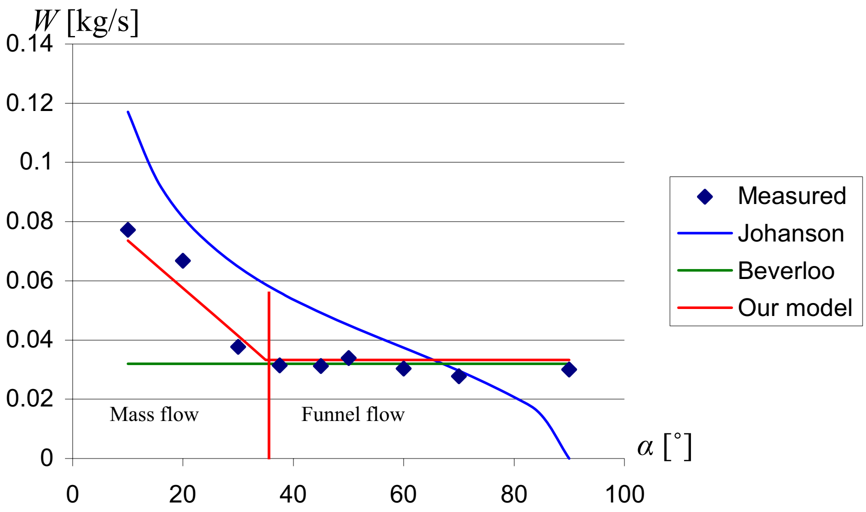


Fig. 13. Measured and predicted discharge rates

In Fig. 13. can be seen that less than 37,5˚ hoppers discharge rate is constant. This is border between mass and funnel flow modes a in Fig. 14. can be seen.

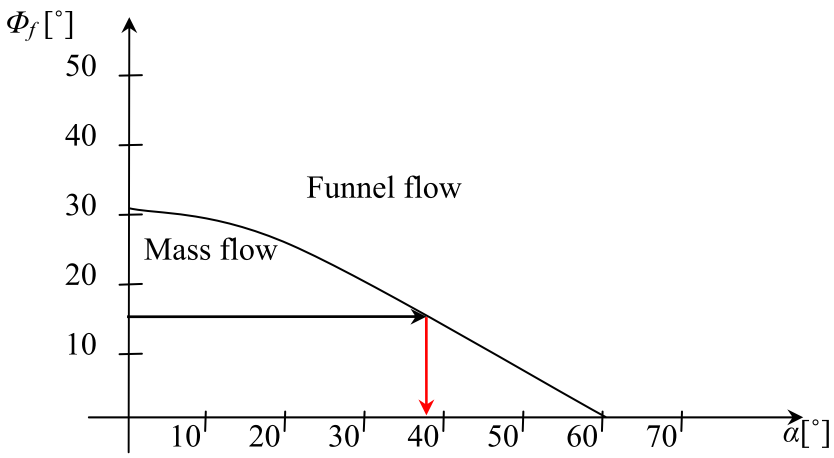


Fig. 14. Changing of flow modes (wheat)

Our model can be used for mass flow empirical if δ is calibrated to cone angle (Table 2.)

6. THESES

1. thesis: Discharge of granular materilas from silos is evaluation and failure of instable arches.

a) Independence of discharge rate from height of heap is conclusion of first thesis

2. thesis: Shape of arches can describe by quadratic function.

3. thesis: At outlet flowing velocity is not constant it can be described by next function:

$$v(x,\varphi) = \sqrt{2g\delta d}\sqrt{\left(1-\left(\frac{2x}{d}\right)^2\right)}$$

Where:
- x: distance from symmetric axis (radius),
- d: outlet diameter,
- g: gravitation acceleration,
- δ: shape coefficient of arch.

4. thesis: Discharge rate of silo can be determined by next computational formula

$$W = \frac{\pi\sqrt{2g}}{6}\sqrt{\delta}\cdot\rho\cdot(d-d_p)^{\frac{5}{2}}$$

Validation limits:

$$7d_p \le d \le 0{,}6D$$

Where:
- d: outlet diameter,
- g: gravitation acceleration,
- ρ: bulk density,
- d_p: diameter of particle,
- δ: shape coefficient of arch,
- D: silo diameter.

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