Methodological Questions in Business Application of the New Product Spread Models
Theses of the doctoral (Ph.D.) dissertation

Lászlóné OROVA

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2010
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PhD school
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NOTATIONS, ABBREVIATIONS

  m potential market size
  p innovation parameter
  q imitation parameter
  t time
  N(t) number of the cumulative adoptions at time t
  OLE Ordinary Least Squares Estimation
  NLLS Nonlinear Least Squares Estimation
  AR(p) auto-regressive model (p represents the order)
  MA(q) Moving-average models (q represents the order)
  ACF autocorrelation function
  PACF partial autocorrelation function
1 Scientific Background, Targeted Objectives

Countries, regions and even the whole world feel great changes in the economy, in the society and in the consumption. The world of the companies and firms are under the pressure of the innovation, generated by the fast technical development, the changing market demand and circumstances, and the global slump in trade of today. Governments expect economic growth and improvement of the international competitiveness from the sponsorship of the innovation processes. The competitiveness has an important role in catching up by the experiments of the OECD countries and the innovation is one of the main influential factors in the rapidly developing countries. The innovation is realized in the complex system of the market demand, infrastructure and institutional framework, firms, education and state research system, innovation policies. There are essentially four types of innovation identified in the Oslo Manual 2005 for measuring innovation: product innovation; process innovation; marketing innovation and organizational innovation.

The quick changes in the economy require the application of objective methods in the planning of the business processes. Those methods are examined in this work, which suppose available time series to model the processes of the future. The application of the objective forecasting methods basing on the data of the past has the obvious background of the assumption of the relative immutability of the business environment but it is required always to plan in the face of the not probable changes in the economy.

The accelerating rates of today’s changes can indicate the irreversible, phenomenal historical progression (historical singularity) that requires the radical paradigm change the supervision of the present ideas, rules. It is not the aim of present study is to examine these global changes and systems.

1.1 The main objectives and problem determination

The main goals of this PhD dissertation are the mathematical modelling of the spreading process of ten new products in Hungary on the time series of the last twenty years; the comparison of these processes to the international results by model parameters; the mathematical sensitivity analyses of the parameters, making forecasts by the model - applying the four goal criteria of the management (Quality – Quantity – Time – Place).

1.1.1 Objectives and Tasks

- The study of the theories of the last fifty-sixty years about the process of the spread of the new products on international scientific sources.
- The study and comparison of the models (especially the mathematical ones) for forecasting market demand and the diffusion of new products, regarding
the data required by the models and the mathematical estimation methods by the Hungarian and the international sources.
- The collection of the data of the new products in Hungary of the last twenty years based on representative samples and the study of data collection methods.
- Choosing the mathematical method applicable on the available data and what can be the base of international comparison in the given period.
- The realization of the computer application of the selected mathematical model and the determination of the parameters of the spread of the new products in Hungary.
- The comparison of the parameters of the new products in Hungary with the international findings.
- The sensitivity analyses of the parameters of the applied mathematical model.
- The search of a mathematical model for short term forecast because the lifecycle time of the products is getting shorter and shorter.

1.1.2 Hypotheses of the PhD dissertation

**H1.** Bass model, which is current in international scientific sources can be applied on Hungarian time series and the parameters of this model are appropriate for the comparison of the diffusion processes of Hungary and other countries.

**H2.** The parameters of the Bass model are dependent of the estimation method, the length and the interval of the time series.

**H3.** A method can be developed to apply Bass model for time series with seasonal effects.

**H4.** Stochastic time series models can be convenient for the short-term forecast of the new product diffusion.
2 Material and Method

2.1 Material
For the investigation of the spreading process of the new products, I collected primary and secondary data. I carried out surveys by paper based, e-mail and WEB-based questionnaires in predetermined target section of the market and I succeeded in getting trade data some times as well. Intended to collect representative data I investigated the time series of consumer durables for hundred households of Hungarian Central Statistical Office, the time series of the National Communications Authority and of the Association and Product Council of Hungarian Mineral Water. The data collection process took place form 2003 to 2009 and some forecasts were controlled by the data of February 2010. The investigated new products are listed in the Table 1.

Table 1. The Investigated New Products – own table

<table>
<thead>
<tr>
<th>New Product</th>
<th>Period</th>
<th>Interval</th>
<th>Source of the Data</th>
<th>Sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>USB storage (pendrive)</td>
<td>2003-2010</td>
<td>yearly</td>
<td>questionnaire</td>
<td>349 students</td>
</tr>
<tr>
<td>MP3 player</td>
<td>2004-2010</td>
<td>yearly</td>
<td>questionnaire</td>
<td>204 students</td>
</tr>
<tr>
<td>Organic bread</td>
<td>1995-2004</td>
<td>yearly</td>
<td>questionnaire (dealers)</td>
<td>60 of Hungary</td>
</tr>
<tr>
<td>Automobile</td>
<td>1960-2010</td>
<td>yearly</td>
<td>HCSO</td>
<td>nationwide</td>
</tr>
<tr>
<td>Video camera</td>
<td>1994-2007</td>
<td>yearly</td>
<td>HCSO</td>
<td>nationwide</td>
</tr>
<tr>
<td>Micro wave oven</td>
<td>1992-2007</td>
<td>yearly</td>
<td>HCSO</td>
<td>nationwide</td>
</tr>
<tr>
<td>Cellular phone</td>
<td>1995-2007</td>
<td>yearly</td>
<td>HCSO</td>
<td>nationwide</td>
</tr>
<tr>
<td>CD player</td>
<td>1995-2007</td>
<td>yearly</td>
<td>HCSO</td>
<td>nationwide</td>
</tr>
<tr>
<td>Cellular phone subscription</td>
<td>1999-2009</td>
<td>monthly</td>
<td>NCA</td>
<td>nationwide</td>
</tr>
<tr>
<td>ISDN subscription</td>
<td>1999-2009</td>
<td>monthly</td>
<td>NCA</td>
<td>nationwide</td>
</tr>
<tr>
<td>Mineral water</td>
<td>1979-2009</td>
<td>yearly</td>
<td>APCHMW</td>
<td>nationwide</td>
</tr>
<tr>
<td>New flavoured alcoholic drink</td>
<td>1995-2003</td>
<td>monthly</td>
<td>Factory delivery</td>
<td>factory</td>
</tr>
<tr>
<td>New packed alcoholic drink</td>
<td>2004-2008</td>
<td>monthly</td>
<td>Trade</td>
<td>nationwide</td>
</tr>
<tr>
<td>New type of tea</td>
<td>2004-2008</td>
<td>monthly</td>
<td>Trade</td>
<td>nationwide</td>
</tr>
</tbody>
</table>

2.2 Method
One of the most widely applied marketing forecast method basing on the data of the past is the analysis of the time series (TS) which consider the trend, period, season and random effects. We can discriminate two types of TS analyses depending on the emphasised effect taken into consideration: one is the deterministic TS, were a predetermined trend is supposed to be modified
more or less by random effects – this is for long term forecasts, the other is the stochastic TS which consider short time effects and emphasise the role of the random effects (Komáromi 2001).

The spread of the innovation is a kind of diffusion process by Rogers(2003), in which an innovation is communicated through certain channels over time among the members of a social system, where innovation is an idea, a practice, or an object that is perceived as new by an individual or other unit of adoption.

In this aspect the new products are also considered as innovation.

I applied two methods to describe the spread of the new product and to make forecasts in this work:

- the deterministic diffusion model of Bass(1969) and
- the stochastic ARIMA model.

Bass(1969) diffusion model is widely influential in marketing and management science, the parameters of this model are the base of the comparisons of the new product diffusion processes not only inside a country, but among countries as well. Despite the considerable application of the Bass(1969) model abroad I have found only one reference to it in Hungarian researches.

There are references to the application of stochastic ARIMA models in the field of the new product diffusion research in the last decade only (Montgomery, Moe 2002, Hassan, and Nassar 2007). As the lifecycle of the products is getting shorter in our accelerating world the importance of the stochastic models grow.

2.2.1 The deterministic Bass(1969) diffusion model

The objective of a diffusion model is to represent the level of spreading of a new product or service among adopters in terms of a simple mathematical function of the lifecycle on the market. The assumptions of the Bass(1969) model: there are no repeat buyers and purchase volume per buyer is one unit; the potential market size is constant in time; innovators buy on the effect of mass-media (external effects) at the beginning and imitators on the effect of the word-of-mouth or other influence from those already using the product (internal effect).

The basic assumption for the Bass(1969) model is that an initial purchase will be made at time \( t \), given that no purchase has been made, and is a linear function of the number of previous buyers. The new adopter is someone from the potential market. The probability of the new adoption is a conditional likelihood this way and can be determined by Bayes’ theorem as follows:

\[
P(t) = \frac{f(t)}{1-F(t)} = p + qF(t),
\]
where

- \( t \) time,
- \( P(t) \) the conditional likelihood, the probability of the new adoption in time \( t \),
- \( f(t) \) density function, unconditional likelihood of the adoption in time \( t \),
- \( F(t) \) distribution function, the probability of the cumulative adoption till time \( t \),
- \( p \) coefficient of innovation (the probability of the first adoption in \( t=0 \); external influence or advertising effect),
- \( q \) coefficient of imitation (internal influence or word-of-mouth effect).

As \( F(T) = \int_{0}^{T} f(t) dt \) and \( F(0) = 0 \), regarding there is no adoption at \( t=0 \), the probability of new adoption in time \( t \) is:

\[
f(t) = (1 - F(t)) \ast (p + q F(t))
\]

If the potential market is \( m \) during the total life-time of the product the cumulative adoptions by time \( t \) are \( N(t) = mF(t) \) and the number of adoptions at time \( t \): \( n(t) = mf(t) \), thus

\[
n(t) = \frac{dN(t)}{dt} = (m - N(t)) \ast \left[ p + q \frac{N(t)}{m} \right] = p(m - N(t)) + q \frac{N(t)}{m} \left[ m - N(t) \right]
\]

Solving the differential equation [1] the cumulative adoptions and the adoptions at time \( t \) are as follows:

\[
N(t) = mF(t) = m \left[ 1 - e^{-(p+q)t} \right], \quad n(t) = mf(t) = m \left[ \frac{p(p + q)}{(p + q e^{-(p+q)t})^2} \right]
\]

If the parameters are known very important information can be calculated for marketing professionals: the time at which the adoption rate reaches its peak \( (t_{\text{max}}) \) and the maximum amount of the adoption \( (n_{\text{max}}) \) if \( p > q \).

\[
t_{\text{max}} = \frac{1}{p + q} \ln(q / p) \quad \text{and} \quad n_{\text{max}} = \frac{m(p + q)^2}{4q}
\]

Bass(1969) advises applying the discrete form [2] of the analogue [1] model because \( Y(T) \) and \( N(T-1) \) are known in time series.
\[ Y(T) = mp + (q - p)N(T - 1) - \frac{q}{m} N(T - 1)^2 \]  

where

\[ Y(T) \quad \text{new adoptions in time interval } T \]
\[ N(T-1) \quad \text{is the cumulative adoptions in } t \leq T-1 \]

if \( m > 0; q-p \geq 0 \) and \( -q/m > 0 \).

The Bass parameters can be estimated by the Ordinary Least Squares Estimation (OLE) method from the equation [3] and I used MS Excel for OLE (trendline – second order polynomial, LINEST built in function, SOLVER, and I wrote VisualBasic procedures as well for the simulation).


I measured the goodness of the fit of the original time series and the estimated series by the square of the Pearson Product Moment Correlation Coefficient (R\(^2\)).

2.2.2 The stochastic ARIMA models

The stochastic time series are usually modelled by the sum or product of trend, seasonal and stochastic components (Rédei, Szentmiklósi 2000):

\[ Y_t = T_t + S_t + \varepsilon_t \]

or

\[ Y_t = T_t \times S_t \times \mu_t \]

where

\[ Y_t \quad \text{observation in the time } t \]
\[ T_t \quad \text{trend component} \]
\[ S_t \quad \text{seasonal component} \]
\[ \varepsilon_t, \mu_t \quad \text{random variables} \]

Stochastic processes

Moving-average (MA) models are stochastic models, which represent the present value of the time series as a function of the present and previous random variables. The general form of an MA(q) model, when the previous \( q \) random effects are taken into consideration, is as follows:

\[ Y_t = \varepsilon_t + \Phi_1 \varepsilon_{t-1} + \Phi_2 \varepsilon_{t-2} + \ldots + \Phi_q \varepsilon_{t-q} \]

where \( q \) denotes the order of the moving-average process.

The auto-regressive (AR) model is a stochastic model, which represents the present value of the time series as a function of its previous values and a variable of the random fluctuation. The general form of an AR(p) model, when the previous \( p \) data of the time series are taken into consideration, is as follows:
\[ Y_t = \theta_1 Y_{t-1} + \theta_2 Y_{t-2} + ... + \theta_p Y_{t-p} + \varepsilon_t \]

where \( p \) represents the order of the auto-regression.

The mixed ARMA(p,q) model represents the present value of the time series in the function of its previous values plus of the present and previous random variables as follows:

\[ Y_t = \theta_1 Y_{t-1} + \theta_2 Y_{t-2} + ... + \theta_p Y_{t-p} + \varepsilon_t + \phi_1 \varepsilon_{t-1} + \phi_2 \varepsilon_{t-2} + ... + \phi_q \varepsilon_{t-q} \]

\[ \text{AR(p)} \quad \text{MA(q)} \]

The seasonal component of the time series can have auto-regression (order \( P \)) and moving average (order \( Q \)) processes as well, thus the model of seasonal time series is as follows: \( \text{SARMA(p, q)(P, Q)} \).

\( Y_t \) variable has only one observed value at new product research. The above mentioned models can be applied only if the time series is stationer. The time series are not usually stationer but they can be transformed to be stationer by transformations such as differencing. The analysis of the original (integrated) time series leads to the \( \text{ARIMA(p,d,q)} \) model, where \( d \) indicates the number of non-seasonal differences. The model for the time series with seasonal and trend effects is \( \text{SARIMA(p,d,q)(P,D,Q)} \), where \( P, D, Q \) are the seasonal parameters.

Estimation, prediction with ARIMA model. Time series modelling has three steps by Box and Jenkins (Ramanathan 2003). First step is the identification that is the determination of \( p, d, \) and \( q \) parameters of the stationer time series. The second step is estimation, when model parameters, \( \theta, \phi \) are estimated, and the third step is the diagnosis, when several tests are run to control if the model fits on the data well. If the fitting is not convenient, an iteration process begins with other, but near parameter values.

I fitted SARIMA models by SPSS and Eviews software.

3 Results

3.1 Bass model on Hungarian time series

The results of my investigation, the Bass parameters of new products launched in Hungary (estimation method: OLE) and the international average values were summarised in Table 2. The coefficient of innovation on yearly data for the new products of Hungary were close to Blute’s (2002) findings in the cases of organic bread and technical products, but it was substantially smaller for the mineral water. The average value of the innovation parameter by Mahajan (2000) was almost three times more than the parameters defined by Blute(2002). The innovation parameters practically equalled a in both American researches. The values of the imitation parameter for the video
recorder and mineral water were very close to the international average value (Mahajan et al. 2000), but for the organic bread it was only the half.

Table 2  Bass parameters – own result: Hungary

<table>
<thead>
<tr>
<th>Data source</th>
<th>New product</th>
<th>Country</th>
<th>Period</th>
<th>p</th>
<th>q</th>
<th>Time series</th>
</tr>
</thead>
<tbody>
<tr>
<td>primary</td>
<td>pendrive</td>
<td>Hungary</td>
<td>06. 2003 – 06. 2005</td>
<td>0.0045</td>
<td>0.2036</td>
<td>monthly</td>
</tr>
<tr>
<td>primary</td>
<td>MP3 device</td>
<td>Hungary</td>
<td>05. 2004 – 12. 2005</td>
<td>0.0038</td>
<td>0.3163</td>
<td>monthly</td>
</tr>
<tr>
<td>primary</td>
<td>organic bread</td>
<td>Hungary</td>
<td>1994-2005</td>
<td>0.013</td>
<td>0.213</td>
<td>yearly</td>
</tr>
<tr>
<td>secondary</td>
<td>automobile</td>
<td>Hungary</td>
<td>1960-1985</td>
<td>0.015</td>
<td>0.1714</td>
<td>yearly</td>
</tr>
<tr>
<td>secondary</td>
<td>Video camera</td>
<td>Hungary</td>
<td>1993 - 2004</td>
<td>0.0157</td>
<td>0.3639</td>
<td>yearly</td>
</tr>
<tr>
<td>secondary</td>
<td>Cellular subscription</td>
<td>Hungary</td>
<td>1st q 1999 – 4th q 2001</td>
<td>0.0166</td>
<td>0.2709</td>
<td>quarterly</td>
</tr>
<tr>
<td>secondary</td>
<td>ISDN subscription</td>
<td>Hungary</td>
<td>1st q 1999 – 4th q 2003</td>
<td>0.024</td>
<td>0.396</td>
<td>quarterly</td>
</tr>
<tr>
<td>secondary</td>
<td>Mineral water</td>
<td>Hungary</td>
<td>1979-2004</td>
<td>0.0051</td>
<td>0.3386</td>
<td>yearly</td>
</tr>
<tr>
<td>secondary</td>
<td>New flavoured alcoholic drink</td>
<td>Hungary</td>
<td>10. 1995 – 10. 2003</td>
<td>0.01294</td>
<td>0.12764</td>
<td>monthly</td>
</tr>
<tr>
<td></td>
<td>Average (Mahajan et al. 2000)</td>
<td>USA</td>
<td>1921-1996</td>
<td>0.04</td>
<td>0.398</td>
<td>yearly</td>
</tr>
<tr>
<td></td>
<td>Average (Blute 2002)</td>
<td>USA</td>
<td>1950-1992</td>
<td>0.01274</td>
<td>0.409</td>
<td>yearly</td>
</tr>
</tbody>
</table>

One of the above investigated new products is pendrive. The Bass model fitted well ($R^2=0.98771$) on the data from the questionnaire. Bass model forecasted in 2005 that only the 62.6% of the students were expected to have pendrive in the future (Figure 1) while the 81% of the students answered the questionnaire to be intended on buying pendrive. The 86% of the students had already pendrive according to my quick control survey in February 2010. The forecast based on the data of the past could not provide correct result for a device of the quickly developing informatics for such a long time, but resulted reasonable one-two year forecasts for the mineral water.
3.2 Sensitivity analyses

3.2.1.1 Methods of estimation

Bass model was fitted by OLE and NLLS on secondary datasets to examine the effect of the method of the estimation on the values of the Bass parameters. The results for the technical new products are summarised in Table 3, and for the foodstuff in Table 4.

Table 3 Bass parameters for technical products – own result

<table>
<thead>
<tr>
<th>Product</th>
<th>Data-source</th>
<th>Period [yearly data]</th>
<th>OLE</th>
<th>NLLS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>p</td>
<td>q</td>
</tr>
<tr>
<td>Automobile</td>
<td>HCSO</td>
<td>1994-2007</td>
<td>0.0049</td>
<td>0.3775</td>
</tr>
<tr>
<td>Video camera</td>
<td>HCSO</td>
<td>1993-2007</td>
<td>0.0134</td>
<td>0.4199</td>
</tr>
<tr>
<td>Personal computer</td>
<td>HCSO</td>
<td>1994-2007**</td>
<td>-0.0236</td>
<td>0.0652</td>
</tr>
<tr>
<td>Microwave oven</td>
<td>HCSO</td>
<td>1992-2007*</td>
<td>0.0428</td>
<td>0.0466</td>
</tr>
<tr>
<td>Cellular phone</td>
<td>HCSO</td>
<td>1995-2007</td>
<td>0.0133</td>
<td>0.7773</td>
</tr>
<tr>
<td>CD player</td>
<td>HCSO</td>
<td>1995-2007*</td>
<td>0.0160</td>
<td>0.1585</td>
</tr>
<tr>
<td>Cellular phone subscription</td>
<td>NCA quick report.</td>
<td>1998-2008*</td>
<td>0.0742</td>
<td>0.2389</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>average</td>
<td>0.0274</td>
</tr>
</tbody>
</table>

Blute (2002) findings for the Bass parameter values of “other countries” were: 0.00378 <=p<= 0.042168 and 0.152295<=q<= 0.535527 (in 90% confidence interval) the parameter values of the Bass model for the investigated new
products launched in Hungary belonged in these intervals, but the goodness of fit is rather bad in more cases according to $R^2$. (The innovation parameter by OLE of the personal computer is not even in the range.)

<table>
<thead>
<tr>
<th>Product</th>
<th>Data-source</th>
<th>Period [yearly data]</th>
<th>OLE</th>
<th>NLLS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$p$</td>
<td>$q$</td>
</tr>
<tr>
<td>Mineral water</td>
<td>APCHMW</td>
<td>1989-2004</td>
<td>0.0051</td>
<td>0.3386</td>
</tr>
<tr>
<td>New flavoured alcoholic drink</td>
<td>Factory delivery</td>
<td>1995-2003</td>
<td>0.0578</td>
<td>0.6220</td>
</tr>
<tr>
<td>New packed alcoholic drink</td>
<td>Trade</td>
<td>2004-2008</td>
<td>0.0463</td>
<td>1.2676</td>
</tr>
<tr>
<td>average</td>
<td></td>
<td></td>
<td>0.0364</td>
<td>0.7427</td>
</tr>
</tbody>
</table>

OLE estimation method was successful on more KSH time series than NLLS and these findings disagreed with Srinivasan and Mason (1986). They found that only NLLS could be applied in certain cases and this estimation method is more effective. If both estimation methods can be applied on the time series the NLLS method gives better fitting. This result agrees with the international experience (e.g. Srinivasan, Mason 1986). The function of the adoption ($Y(t)$) fits not so good than the cumulative adoption function ($N(t)$) in the case of both estimation methods. The innovation parameters of the NLLS were significantly less, the imitation parameters a little more than of the OLE – this agrees with the international results.

### 3.2.1.2 Examination of the length of the time series

**Adding data to the end of the data series**

Data series of cellular phone and ISDN subscriptions were investigated to reveal how the Bass parameters depend on the length of the time series.
### Figure 2  Sensitivity investigation – own result

Empirical regularities were found during the estimation of Bass model: increasing the number of the data of the time series that means the cumulative adoption, the estimated parameters changed as follows: the innovation parameter increased, the imitation decreased. The market potential for example increased by 8% (5% Blute and Lilien) at the ISDN data, imitation parameter by 15% (15% Blute and Lilien) and the innovation parameter decreased by 18% (10%). Blute and Lilien used NLLS regression and me OLS, but the found regularity depends on the model with an unknown ceiling and not on the regression method.

**Removing data from the beginning of the data series**

The examination of the effect on Bass parameters of the removing data from the beginning of the data series was carried out on simulated data series, because the investigation on empirical data had no result - the values of the time series fluctuated. The simulated time series were generated on the average Bass parameters of the international results.

Taking away the first 23 data of the time series one at a time, the Bass parameters, the estimated value of the potential market, the goodness of the fit of the second order function ($R^2_{so}$) at OLE and the goodness of the fit of the adoption and cumulative adoption functions ($R^2_y$, $R^2_N$) were counted by my Visual Basic program. I realised that the data taken away from the beginning of the data series did not modified so much the Bass parameters at slowly increasing time series than in the case of the strongly increasing time series and the goodness of fit ($R^2_y$) augments at the first case. The absence of 5-10 data from the beginning of the data series resulted bias especially in the value of the estimated potential market at each data series (Figure 3).
I found that the innovation parameter increases and the imitation parameter decreases if data are taken away from the beginning of the data series or data are added to it.

The start data of the time series

It frequently occurs that the new product is adopted hardly at the beginning, the curve of the adoption increases slowly and the Bass model does not fit. The start of the simulated time series (Mahajan et al. (2000): p=0.037; q=0.327) was lengthened by ascendant data of the linear function of the time. The possibility of estimating the Bass parameters were investigated in relationship with the number of the added data. Estimation method was OLE and the goodness of the fit was not taken into consideration. The model could be fitted (OLE) at minimum 3% increase of the data at simulated time series, but 2-2.5 times more increase was needed at empirical time series because they fluctuate well. Bass suggested that the observed time series should have minimum 10% increase.

3.2.1.3 Investigation of time series of different intervals

Bass model was fitted on monthly, quarterly and yearly data to determine the model parameters and to forecast the potential model size by: a) OLS on the discrete form of the basic Bass model and b) NLLS on the Y(t) from the analogue Bass model in the cases of cellular phone subscription, new flavoured and new packed alcoholic drinks. (The observations of the different fitting methods corresponded with the results discussed above.) The quarterly and yearly data series were calculated from the monthly data. The investigation results of the new flavoured alcoholic drink was summarised in Table 5.
Comparing the results of the time series with different intervals obvious bias in Bass parameters (p, q) appeared without reference to the fitting method. Correlation analysis was carried out on the time interval (t), p, q, m and $R^2$ at both regression methods for two new products (new flavoured and new packed alcoholic drink). The parameters of significant correlations were summed up in Table 6.

Investigation of the correlations of the Bass parameters was extended on simulated data series as well. Yearly time series were created on empirical Bass parameters (Mahajan at al. (2000) average parameters on long, short data series, and Talukdar (2002) for developed countries). Data series of two and four years intervals were made from the data of the yearly data series and Bass model was fitted on the three types of time series by OLE and NLLS. Bass parameters (p and q) were the linear function of the interval ($R^2 \geq 0.976$) in each case. Figure 4 shows the experience in the simulated time series of Mahajan.
The parameters of times series of half-year interval were estimated by the found linear relation applying the parameters of the NLLS estimation method. The goodness of the fitting of the estimated and base data resulted $R^2 = 0.877083$ thus the parameters from the extrapolation process capable of practical application.

The only reference I found investigating times series with different intervals (Wright, Uritchard, Lewis 1997) did not find relationship between the interval of the time series and the Bass parameters.

### 3.3 Investigation of seasonal times series by Bass model

The delivered quantity of the new flavoured alcoholic drink from the factory showed seasonal effect: it was high before Christmas and new year’s eves, it decreased after this period and was restrained during summers. Bass model not handling the seasonality provided inaccuracy in the forecast. Fitting the Bass model separately on the data of the first, second, third and fourth quarter it resulted model parameters for the definite quarter, which was the base of the accurate quarterly forecast. The forecast bias was less then 5% applying the model on six from eight base data and comparing the forecast to the less two base data. Figure 5 shows that trends of the models on the total and separated quarterly data were the same.
3.4 The stochastic ARIMA model

The investigated new type of tea was a line extension regarding the type of the new product. The plot of the monthly sales did not show Roger’s bell-shaped diffusion process, thus, there must have been some marketing efforts in the background. This could resolve why Bass(1969) diffusion model could not have been fitted on these data. The ARIMA model could be fitted, since time series models are empirical and flexible and are not based on certain restrictive assumptions about the process (Hassan and Nassar 2002). The ARIMA(1,0,2) model gave the best fitting on this time series ($R^2$ is 0.71 between the observed and fitted data). This result showed that the purchase of the new type of tea in a given month depended a great deal on the amount of the purchase of the previous month ($\Theta_1 = 0.76$) and on the random deviation of the last two months.

The alcoholic drink was a new category entry as it was a new-to-the company product and new-to-the market product in Hungary as well. However, it was not new to the general market, as it existed on the market abroad. The identification process of ARIMA recommended to fit the ARIMA(2,1,2) model, and the plot of the data series to fit the SARIMA(2,1,2)(0,12,0) model. It was interesting that the purchase of the previous month had an increasing effect on the amount of the purchase of the current month, but 2 month prior had a regressive effect as $\Theta_1 > 0$ and $\Theta_2 < 0$ in both models. Figure 6 provides one-year forecast for the sales volume of the alcoholic drink by SARIMA(2,1,2)(0,12,0)
Figure 6   Forecast of the New Packed Alcoholic Drink Model: \( \text{SARIMA}(2,1,2)(0,12,0) \) – own result

The ARIMA model provided better approximation than Bass model on each time series: e.g. investigating the time series of the new packed alcoholic drink \( R^2=0.66 \) and \( 0.67 \) in the case of OLE and NLLS, but \( R^2=0.856 \) by ARIMA\((2,1,2)\). SARIMA\((0,1,1)(1,12,1)\) model fitted the best on the data series of the cellular phone subscription. The model parameters were: \( MA=0.71334336; \) \( SAR1=0.29350874; \) \( SMA1=0.83845321; \) constant= -0.002 regarding the time series until the end of the year 2008.

Figure 7   Forecast of the Monthly Change of the Subscription of Cellular Phones of 100 Inhabitants – own result A: base data, B: fitted values, C and D: one and two years control data
Figure 7 shows forecast and the effective values until the January of 2010. The model overestimates in the two year forecast.

Table 7  Summary of the ARIMA Models in the New Product Diffusion

<table>
<thead>
<tr>
<th>New product</th>
<th>Country</th>
<th>Model</th>
<th>Data collection time</th>
<th>Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wireless telephone subscribers*</td>
<td>USA</td>
<td>AR(3)</td>
<td>1950-1993</td>
<td>yearly</td>
</tr>
<tr>
<td>Album**</td>
<td>USA</td>
<td>ARIMA(1,0,1)</td>
<td>1993-95 (13 time series)</td>
<td>weekly</td>
</tr>
<tr>
<td>Cellular phone subscribers</td>
<td>Hungary</td>
<td>SARIMA(0,1,1)(1,12,1)</td>
<td>1999-2008</td>
<td>monthly</td>
</tr>
<tr>
<td>New flavoured alcoholic drink</td>
<td>Hungary</td>
<td>SARIMA(2,0,2)(0,12,0)</td>
<td>1995-2003</td>
<td>monthly</td>
</tr>
<tr>
<td>New type of tea</td>
<td>Hungary</td>
<td>ARIMA(1,0,2)</td>
<td>2003-2007</td>
<td>monthly</td>
</tr>
<tr>
<td>Alcoholic drink in new package</td>
<td>Hungary</td>
<td>SARIMA(2,1,2)(0,12,0)</td>
<td>2004-2008</td>
<td>monthly</td>
</tr>
</tbody>
</table>

* Hassan, Nassan(2007);** Montgomery, Moe(200); Hungarian – own result

Table 7 summarises the aforementioned ARIMA models of the new product diffusion. The investigations of new alcoholic drinks showed some similarity: the purchase and the random effect of the two last months determine the current purchase in both case of alcoholic drinks (AR(2), MA(2)). Models showed that the time series of the cellular phone subscription and the new packed alcoholic drink had trends and it means that they were very successful new products in Hungary.

Comparing my results to the international experience great random effect can be found in Hungary (MA(1-2) and the model parameter is large, $\Phi_i \geq 0.5$ almost in each case. There is no or just one period long random affect in the case of the two foreign observations.

American research shows that the album sales had a large jump at the beginning, but showed an exponential declining pattern later. The album is a type of fashion product – it can not be compared to the investigated Hungarian new products; while the wireless phone shows a very slow raise due to the technical development, which was going on that time.

3.5 New and novel scientific results

1. Bass model is convenient for the new products in Hungary, if the time series shows the characteristic of the Rogers’ adoption curve. I followed up my research on the time series of fourteen new products in Hungary between 1965 and 2009. The model was applied for foodstuffs and services beyond consumer durables with the extension of the interpretation of the “inner effect” in the
period of market growth. I suggested a new method for the utilisation of the Bass model on seasonal time series and I made seasonal forecasts in this way.

2. I observed that the innovation parameter of the Bass model is less than the imitation parameter by an order of magnitude and it is sensitive to the type of the product and to the economic position of the country by the international sources. I diagnosed that Hungarian customers are moderate innovative, but they buy the products considered convenient by them because the imitative parameter did not lag behind the international average value, it is even higher in the cases of the drinks.

3. I carried out an investigation regarding the sensitivity of the Bass model from three viewpoints: estimation methods, the relationship between the parameters and the interval of the time series and the selection of the start point of the time series under examination.

   *Estimation method:* I completed the statement of Srinivasan and Mason (1986) - NLLS can only be utilised in certain cases and this estimation method is more effective – with that there are cases when OLE method - basing on the discrete form of the model – is successful, thus it is always recommended to try OLE method if the NLLS is not successful. I found that the adoption function \( Y(t) \) fits worse than the cumulative adoption curve \( N(t) \) in the both cases of the estimation methods. I found that the innovation parameter is less and the imitation parameter is bigger in the case of NLLS estimation than of OLE estimation method examining the same time series, but I did not found it as much as an order of magnitude.

   *The interval of the time series:* Correlation can be detected between the Bass parameters on simulated time series. This relation was detected on empirical data series as well. I suggested a parameter conversion method for bisected interval with precision acceptable in practice.

   *The start data of the time series:* The Bass parameters of the simulated time series shortened at the beginning are close to the real values in the case of slowly increasing time series, but significant deviations in the parameters were detected discarding some values from the beginning of the rapidly increasing time series. I gave estimation for the minimum average increment of the time series with long beginning part to be able to fit the model.

4. I found that the stochastic ARIMA model, which is widely used on other fields can be applied on time series with seasonal effects, or on time series with or without the characteristics of the Rogers adoption curve to forecast the adoption of new products.

4 **Conclusions and Recommendations**

I fitted the deterministic Bass model on 13 time series successfully. The time series contained the data of about the last twenty years, not regarding the data of the automobile. I compared the new product diffusion process in Hungary
and abroad by the parameters of the Bass model because these parameters are known for several countries. I found only one reference to Bass parameters for a new product in Hungary, it was the mineral water (Sipos 2009). I confirmed the H1 hypothesis and I remarked that the long-term forecast of the Bass model should be treated with prudence.

I fitted the Bass model by two estimation methods: the discrete form of the model was fitted by OLE and the continuous by NLLS. The Bass parameter showed deviation on same time series: a) the innovation parameter is less and the imitation parameter is greater by some percentages in the case of the NLLS than of the OLE, b) NLLS gives the better fitting regarding $R^2$ what corresponded with the results of the international sources. The function of the adoption, $Y(t)$ fitted worse than the function of the cumulative adoption, $N(t)$ in both cases of the estimation methods. The Bass parameters of either empirical or simulated time series of different intervals showed correlation. This relation can be the base of the comparison of the Bass parameters of the time series with different intervals, what stands in need of the practice since the examination of the time series with shorter intervals are recommended by research sources (Putsis, 1996) - I suggested a method for it in my work. The innovation parameter increased, the imitation parameter decreased examining the latter part of the time series in my observations. I confirmed the H2 hypothesis and I made the conclusion that Bass parameters can be the base of the comparison of the diffusion processes if the estimation method and the interval of the time series are the same, I propose the indication of these data in scientific sources this way. I completed H2 hypothesis: there is correlation between the Bass parameters and innovation parameter is more sensitive to the estimation method than the imitation parameter.

I applied the Bass model separately by seasons on seasonal time series and made forecast for the seasons respectively, I confirmed the H3 hypothesis this way and completed with that the innovation parameter differ more than imitation parameter by seasons. I worked out the application of the Bass model on seasonal time series.

I applied the stochastic ARIMA model on four time series with seasonal effects, with or without the characteristics of the Rogers adoption curve as well. The short term forecasts of the model has great importance in practice, I confirmed the H4 hypothesis.

**Proposals for Practical Application**

- I propose the indication of the interval of the time series and the estimation method in scientific sources.
- I propose my method for the application of the Bass model on seasonal time series.
- I propose ARIMA model and other forecasting techniques for short time forecasting and for comparative analysis in our accelerating world.
I came to know during my work that firms are disinclined to provide data for university research alluding to misappropriation of trade secrets. Referring to the importance of the relationship between the university and the trade in the knowledge based economy and society I propose to establish a forum where trade data are available for university researches considering the benefits of the bilateral research.

5 Publications of the Author in this Topic

a) Scientific publications (books, book parts)

**Book part in Hungarian**

b) Scientific articles

**In English**


**In Hungarian**

Komáromi A. - **Orova L.-né**: Matematikai modellek az innováció terjedésében, Szakmai Füzetek Budapesti Gazdasági Főiskola Klíkereskedelmi Főiskolai Kar, 19. szám, 2007. 94-100. pp. ISSN 1587 5881

**Orova L.-né**: Kutatói feladat adatgyűjtésének internetes támogatása. ACTA Agrária Kaposváriensis. Volume 10 No 3 2006. 75-88. pp. ISSN 1418-1789


c) Scientific conference lectures in research issues

**In English**

In Hungarian


f) Course notes in Hungarian


Oktatott tantárgyak honlapjai


Citation
